General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MFP3**
Unit Further Pure 3

Friday 25 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x^2 - y^2$$

and $$y(2) = 1$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

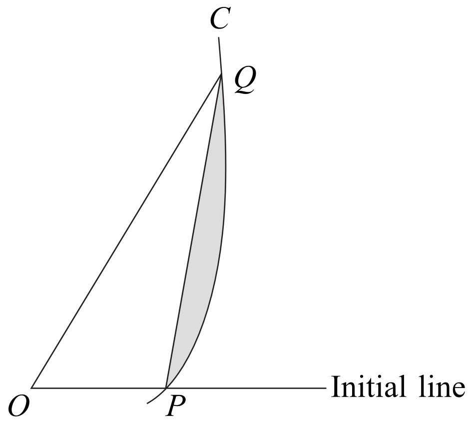
with $h = 0.1$, to obtain an approximation to $y(2.1)$. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to $y(2.2)$. *(3 marks)*

2 The diagram shows a sketch of part of the curve C whose polar equation is $r = 1 + \tan\theta$. The point O is the pole.



The points P and Q on the curve are given by $\theta = 0$ and $\theta = \frac{\pi}{3}$ respectively.

(a) Show that the area of the region bounded by the curve C and the lines OP and OQ is

$$\frac{1}{2}\sqrt{3} + \ln 2$$ *(6 marks)*

(b) Hence find the area of the shaded region bounded by the line PQ and the arc PQ of C. *(3 marks)*

3 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 5$$ *(6 marks)*

(b) Hence express y in terms of x, given that $y = 2$ and $\frac{dy}{dx} = 3$ when $x = 0$. *(4 marks)*

4 (a) Explain why $\int_1^{\infty} x e^{-3x}\, dx$ is an improper integral. *(1 mark)*

(b) Find $\int x e^{-3x}\, dx$. *(3 marks)*

(c) Hence evaluate $\int_1^{\infty} x e^{-3x}\, dx$, showing the limiting process used. *(3 marks)*

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5 By using an integrating factor, find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{4x}{x^2+1}y = x$$

given that $y = 1$ when $x = 0$. Give your answer in the form $y = \mathrm{f}(x)$. *(9 marks)*

6 A curve C has polar equation

$$r^2 \sin 2\theta = 8$$

(a) Find the cartesian equation of C in the form $y = \mathrm{f}(x)$. *(3 marks)*

(b) Sketch the curve C. *(1 mark)*

(c) The line with polar equation $r = 2\sec\theta$ intersects C at the point A. Find the polar coordinates of A. *(4 marks)*

7 (a) (i) Write down the expansion of $\ln(1+2x)$ in ascending powers of x up to and including the term in x^3. *(2 marks)*

(ii) State the range of values of x for which this expansion is valid. *(1 mark)*

(b) (i) Given that $y = \ln\cos x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$. *(4 marks)*

(ii) Find the value of $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ when $x = 0$. *(3 marks)*

(iii) Hence, by using Maclaurin's theorem, show that the first two non-zero terms in the expansion, in ascending powers of x, of $\ln\cos x$ are

$$-\frac{x^2}{2} - \frac{x^4}{12}$$

(2 marks)

(c) Find

$$\lim_{x\to 0}\left[\frac{x\ln(1+2x)}{x^2 - \ln\cos x}\right]$$

(3 marks)

8 (a) Given that $x = \mathrm{e}^t$ and that y is a function of x, show that:

(i) $x\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t}$; *(3 marks)*

(ii) $x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} - \dfrac{\mathrm{d}y}{\mathrm{d}t}$. *(3 marks)*

(b) Hence find the general solution of the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6x\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 0$$ *(5 marks)*

END OF QUESTIONS

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